Furthermore, for a different problem, with two panels and an imposed pressure gradient, there are regions between the maximum and minimum transition position that are not accessible and other positions for which a solution exists but to which the algorithm will not converge. When the optimum occurs at a discontinuity in the solution surface, the method is bound to fail because the gradient estimation step involves constructing a local linear model of the relationship between the suction flow rates and the transition position. Of course, any solution scheme that relies on local smoothness of the underlying data will also fail in such a condition.

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Associate Editor

Channel Flow Instability in Presence of Weak Distributed Surface Suction

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Nomenclature

c_i = amplification rate of Tollmien–Schlichting waves
c_r = phase speed of flow disturbances
i = imaginary unit
p_0 = pressure associated with the basic flow
p_1 = pressure associated with flow disturbances
Re = Reynolds number based on the half-channel height
S_L (S_U) = amplitude of suction wave at the lower (upper) wall
t = time
(u, v) = amplitude function associated with flow disturbances
(u_1, v_1) = velocity vector associated with flow disturbances
$\|(u, v)\|$ = energy norm associated with flow disturbances; Eq. (6)
$\boldsymbol{V}$ = total velocity vector
$\boldsymbol{V}_0$ = velocity vector associated with the basic flow, (u_0, v_0)
v_L (v_U) = suction velocity at the lower (upper) wall
x = streamwise coordinate
y = normal-to-the-wall coordinate
z = spanwise coordinate
α = wave number of wall suction and flow disturbances

I. Introduction

UNIFORM surface suction is an accepted tool for stabilization of laminar boundary layers. In applications the suction distribution is not likely to be constant. Thus, one must consider flow instability in the presence of suction nonuniformities. Whereas spatial distribution of these nonuniformities cannot be predicted apriori, one can assume that their amplitude is of the same order as any other environmental disturbances, as well as of the same order as Tollmien–Schlichting (TS) waves. It is not known how the presence of these nonuniformities may affect flow behavior.

Recently Floryan[1] demonstrated that suction nonuniformities of sufficiently large amplitude induce an instability that manifests itself through generation of streamwise vortices. This instability represents a bypass route to transition that does not rely on the growth of two-dimensional TS waves during its early stages.[2] The formulation given in Ref. 1 assumes suction-induced flow modifications to be larger than flow disturbances, and thus, it cannot resolve issues being addressed in the present study.

The present work is focused on the analysis of the behavior of two-dimensional TS waves in the presence of suction nonuniformities that induce flow modifications of the magnitude comparable to the magnitude of the TS waves. The spatial pattern of the nonuniformities can be either fixed or moving. Because of the linearity of the problem, it is sufficient to consider suction in the form of a single Fourier harmonic and to investigate the effects of variations of its wave number and phase speed. The goals of this work are to determine 1) whether suction nonuniformities affect neutral stability conditions and 2) what type of suction nonuniformities may interfere with the TS waves. The analysis is carried out in the context of plane Poiseuille flow, which represents a standard reference case.

II. Problem Formulation

Consider plane Poiseuille flow confined between flat rigid walls at $y = \pm 1$ and extending to $\pm\infty$ in the x and z directions (Fig. 1). The velocity and pressure fields have the form

$$\boldsymbol{V}_0(x, y) = [u_0(y), 0] = [1 - y^2, 0], \qquad p_0(x, y) = -2x/Re \tag{1}$$

where the fluid motion is directed toward the positive x axis and the Reynolds number Re is based on the half-channel height and the maximum x velocity. At the upper and lower walls, apply suction in the form

$$u_1(x, \pm 1, t) = 0$$
$$v_1(x, -1, t) = v_L(x, t) = \tfrac{1}{2} S_L e^{i\alpha(x - c_r t)} + \mathrm{CC}$$
$$v_1(x, +1, t) = v_U(x, t) = \tfrac{1}{2} S_U e^{i\alpha(x - c_r t)} + \mathrm{CC} \tag{2}$$

where c_r and α denote the phase speed and the wave number of the suction wave, respectively, and CC is the complex conjugate. Because the problem is linear, the response of the flow to an arbitrary suction can be determined by considering only two cases: 1) $S_L = S_U \equiv (S, 0)$, symmetric suction, and 2) $S_U = -S_L \equiv (S, 0)$, asymmetric suction, where S is real. The velocity and pressure fields can be represented as

$$\boldsymbol{V}(x, y, t) = [u_0(y), 0] + [u_1(x, y, t), v_1(x, y, t)]$$
$$p(x, y, t) = p_0(x) + p_1(x, y, t) \tag{3}$$

where (u_1, v_1) and p_1 are the velocity and pressure modifications due to the presence of wall suction. We seek a solution in the

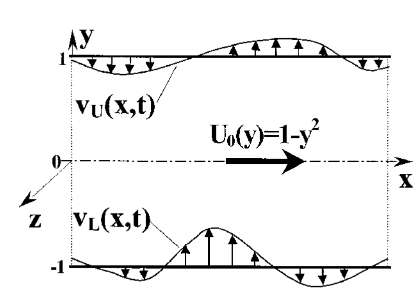

Fig. 1 Flow configuration.

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form $q_1(x, y, t) = q(y)\exp[i\alpha(x - c_r t) + \alpha c_i t] + \mathrm{CC}$, where q_1 is any flow quantity and α, c_r, and c_i are real. Steady flow modifications induced by the suction correspond to $c_i = 0$, whereas growing, neutral, and decaying TS waves correspond to $c_i > 0$, $c_i = 0$, and $c_i < 0$, respectively. Substitution of the assumed form of the solution into the linearized Navier–Stokes and continuity equations results in

$$(-i\alpha c_r + \alpha c_i)u + i\alpha u_0 u + v\mathrm{D}u_0$$
$$= -i\alpha p + (1/Re)(-\alpha^2 u + \mathrm{D}^2 u)$$
$$(-i\alpha c_r + \alpha c_i)v + i\alpha u_0 v = -\mathrm{D}p + (1/Re)(-\alpha^2 v + \mathrm{D}^2 v)$$
$$i\alpha u + \mathrm{D}v = 0 \tag{4}$$

where $\mathrm{D} = \mathrm{d}/\mathrm{d}y$, with boundary conditions in the form

$$u(\pm 1) = 0, \qquad v(-1) = \tfrac{1}{2}S_L, \qquad v(+1) = \tfrac{1}{2}S_U \tag{5}$$

when $c_i = 0$. Only homogeneous boundary conditions are admissible when $c_i \neq 0$.

III. Discussion

Equations (4) with boundary conditions (5) form an inhomogeneous boundary value problem that describes flow modifications induced by the suction. Equations (4) supplemented with homogeneous boundary conditions describe TS waves. We shall demonstrate that both classes of solutions may interfere with each other only when $c_i = 0$.

The homogeneous problem forms an eigenvalue problem whose nontrivial solution exists only for certain combinations of α, c_r, c_i, and Reynolds number. The required dispersion relation has to be determined numerically. Figure 2 shows the dispersion relation for the first symmetric eigenmode that represents the dominant TS wave.

Figure 3 presents the magnitude of the inhomogeneous solution (e.g., the magnitude of the flow modifications associated with the suction) defined as the norm

$$\|(u, v)\| := \left\{\int_{-1}^{1} |v(y)|^2\,\mathrm{d}y + \int_{-1}^{1} |u(y)|^2\,\mathrm{d}y\right\}^{\frac{1}{2}} \tag{6}$$

as a function of α and c_r for the supercritical value of the Reynolds number $Re = 10^4$ and for the symmetric and asymmetric suctions. The preceding norm is directly related to the kinetic energy of flow modifications. In other words, kinetic energy of the flowfield described by velocity vector ($u\exp[i\alpha(-c_r t)] + \mathrm{CC}$, $v\exp[i\alpha(-c_r t)] + \mathrm{CC}$) and calculated over one period $\lambda_x = 2\pi/\alpha$ is equal to $\lambda_x\|(u, v)\|^2$. Calculations have been carried out with machine accuracy using the methodology based on spectral discretization described in Ref. 3. It can be seen that the flow response to symmetric suction is infinite for two suction waves, which we shall refer to as the critical waves, and very large for suction waves with characteristics close to the critical waves. Infinite response signals critical stability conditions. Comparison of Figs. 2 and 3 shows that characteristics of the critical surface suction waves are exactly the same as characteristics of the neutral TS waves.

The character of the flow response to critical suction waves is well illustrated in Fig. 4 for $\alpha = 0.797$ and 1.095, $Re = 10^5$. For these conditions, two symmetric neutral TS waves with $c_r \approx 0.213$ and 0.247, respectively, exist. To explain this response, recall that solution of the inhomogeneous problem exists if and only if the inhomogeneity is orthogonal to all solutions of the adjoint homogeneous problem (Fredholm alternative). When the suction is symmetric, the associated inhomogeneity cannot, in general, be orthogonal to the solutions of the adjoint homogeneous problem. Thus, the solution of the inhomogeneous problem cannot exist. This is manifested by the solution becoming infinite (Fig. 4). This effect may be interpreted as a resonance between the neutral TS wave and the (critical) surface suction wave. The effect of the resonance decreases rapidly with an increase in the detuning between both waves. When the suction is asymmetric, the (asymmetric) inhomogeneity is orthogonal to all symmetric solutions of the adjoint problem, and thus a solution may exist. The flow response consists of a nonunique symmetric

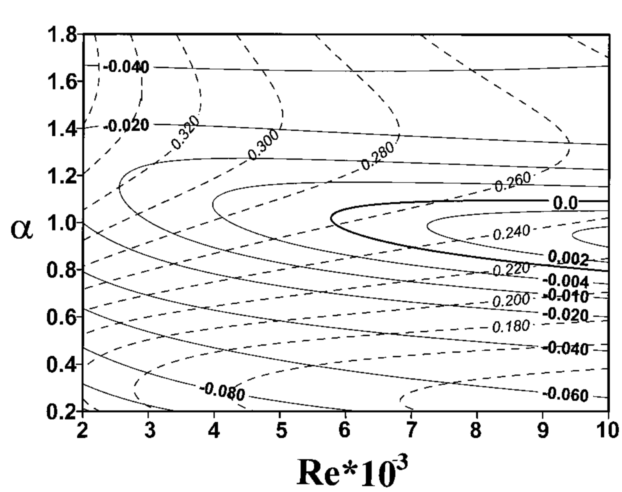

Fig. 2 Stability diagram describing the first symmetric mode of the Poiseuille flow; continuous and dashed lines correspond to c_r and c_i, respectively.

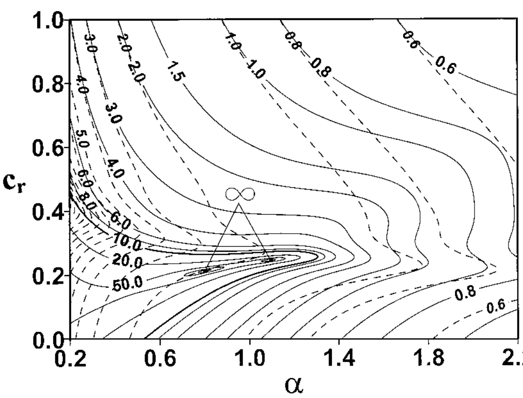

Fig. 3 Variations of the magnitude of the flow response $\|(u, v)\|$ as a function of the suction wave number α and wave speed c_r for $Re = 10^4$; continuous and dashed lines correspond to the symmetric and asymmetric suctions, respectively.

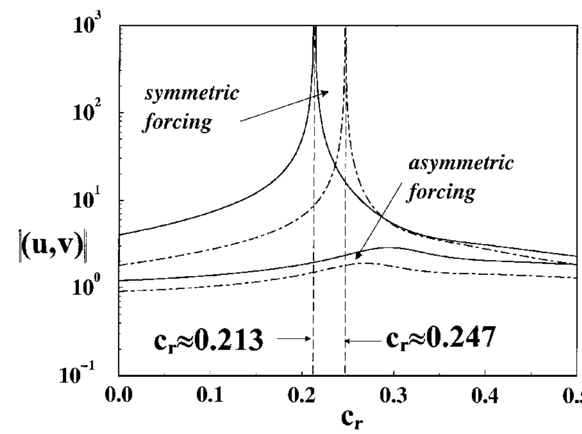

Fig. 4 Variations of the magnitude of the flow response $\|(u, v)\|$ as a function of c_r. The neutral TS waves exist for $(\alpha, c_r) \approx (0.797, 0.213)$ and $(1.095, 0.247)$; continuous and dashed lines correspond to suction wave numbers $\alpha = 0.797$ and 1.095, respectively.

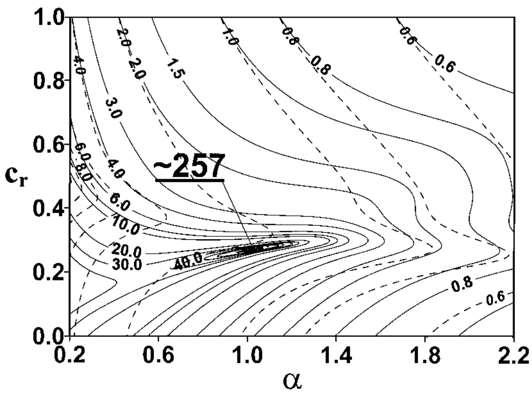

Fig. 5 Variations of the magnitude of the flow response $\|(u, v)\|$ as a function of the suction wave number α and wave speed c_r for $Re = 5 \times 10^3$; continuous and dashed lines correspond to the symmetric and asymmetric suctions, respectively.

eigensolution (TS wave) and a unique asymmetric particular solution of the inhomogeneous problem. A very large response of the flow is possible in this case but requires a perfect tuning between the neutral TS wave and the surface suction wave.

Results presented in Fig. 3 demonstrate that linear theory is inadequate in the neighborhood of (linear) neutral stability points and a nonlinear theory must be used regardless of the magnitude of the suction. The nonlinear effect may expand the range of the TS instability and may initiate an instability of the type studied in Ref. 1. Figure 5 shows flow response to surface suction for the subcritical value of Reynolds number $Re = 5 \times 10^3$ when all TS waves are stable. The near resonance between the TS and the surface suction waves may lower the critical Reynolds number due to the subharmonic character of the instability, provided that suction with sufficiently large amplitude is used.

IV. Conclusions

The preceding results demonstrate that small suction nonuniformities may affect the flow instability only if they contain the critical or near-critical suction waves, that is, waves with the same or almost the same wave number α and phase speed c_r as the neutral TS waves. Suction waves slightly detuned with the neutral TS waves may affect instability provided that the amplitude of suction nonuniformities is large enough. Our conclusions could be readily tested in a flow with a few suction slots at the wall that could emulate suction waves with the desired wave number and phase speed.

Acknowledgments

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Vibration of Thermally Stressed Pretwisted Cantilever Composite Plates

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I. Introduction

PRETWISTED cantilever composite plates are used extensively as blades in engines and turbomachinery and are very important structural members. Therefore their vibration characteristics are important for efficient sizing and design. Composite pretwisted plates are integral parts of engines of high-performance aircraft and naval structures. These are envisioned to operate at very high or low temperatures. Supersonic and hypersonic flight can be noted in this regard. This applies equally well to other structures operating in hostile environments. It thus appears imperative to study the vibration characteristics of pretwisted composite plates that are thermally stressed.

Studies concerning the vibration of blades as well as isotropic and composite twisted cantilever plates have appeared in the literature.[1–5] In addition, researchers are studying the free vibration of composite laminated plates subjected to temperature changes.[6–8] The purpose of the present work is to study the variation of the fundamental natural frequency for pretwisted cantilever composite plates for various laminations at different temperatures and angles of twist.

II. Computational Experiments

Figure 1 illustrates the composite pretwisted cantilever plate; also shown are the geometric and material data. Two symmetric and two asymmetric eight-layer laminations are considered, namely, 1) $(0/90/0/90)_s$, 2) $(45/-45/0/90)_s$, 3) $(30/-30)_4$, and

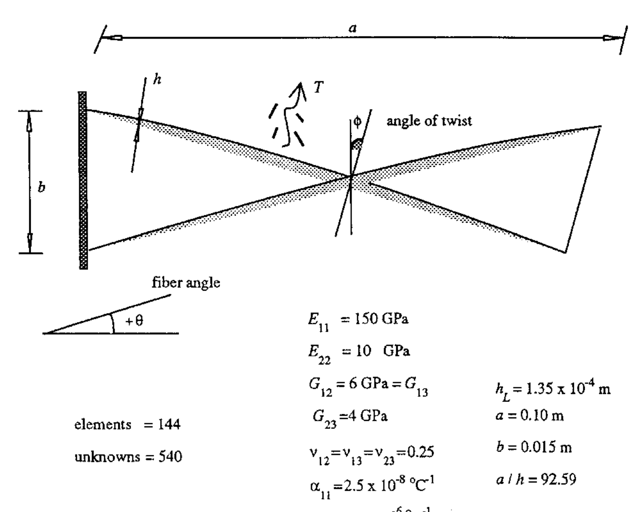

Fig. 1 Geometrical and material data for a pretwisted cantilever composite plate.

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